• Analysis •

1. Correct Answer: $1\frac{5}{12}$

⊙ **Sample Solutions:**

1.

$$^2\cancel{3}\frac{1}{6} = \frac{12}{12} + \frac{2}{12} = \frac{14}{12}$$
$$-\ 1\frac{3}{4} \qquad\qquad \frac{9}{12}$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$1\frac{5}{12}$$

2.

$$^2\cancel{3}\frac{1}{6} = \frac{24}{24} + \frac{4}{24} = \frac{28}{24}$$
$$-\ 1\frac{3}{4} \qquad\qquad \frac{18}{24}$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$1 \qquad\qquad\qquad \frac{10}{24} = 1\frac{5}{12}$$

⊙ The student needed to know how to subtract mixed numbers that require regrouping (borrowing). This includes all of the following:

  1. Finding a common denominator

  2. Regrouping (borrowing) from the whole number

  3. Subtracting the fractions and subtracting the whole numbers

  4. Simplifying the answer, if necessary (not needed in this particular problem)

  SKILL: Subtract mixed numbers with unlike denominators and simplify the answer to lowest terms.

2. Correct Answer: 2

⊙ This problem could be solved in one of the following three ways.

$$2\frac{1}{4} \div 1\frac{1}{8} =$$

$$\frac{9}{4} \div \frac{9}{8} =$$

$$\frac{\cancel{9}}{\cancel{4}_1} \cdot \frac{\cancel{8}^2}{\cancel{9}} = \frac{2}{1} = 2$$

⊙ **Solution A:**

  1. Change the mixed numbers to improper fractions.

  2. Show to multiply by the reciprocal of the second factor.

  3. Divide the common factors.

  4. Multiply the numerators and denominators.

○ **Solution B:**

1. Change the mixed numbers to improper fractions.
2. Multiply by the reciprocal of the second factor.
3. Divide the answer by the Greatest Common Factor (GCF) or a common factor.

$$2\frac{1}{4} \div 1\frac{1}{8} =$$

$$\frac{9}{4} \div \frac{9}{8} =$$

$$\frac{9}{4} \cdot \frac{8}{9} = \frac{72}{36} = 2$$

○ **Solution C:**

1. Change the mixed numbers to improper fractions.
2. Find a common denominator.
3. Divide the numerators and then divide the denominators.

$$2\frac{1}{4} \div 1\frac{1}{8} =$$

$$\frac{9}{4} \div \frac{9}{8} = \frac{18}{8} \div \frac{9}{8} = \frac{18 \div 9}{1} = \frac{2}{1} = 2$$

SKILL: Divide mixed numbers and simplify the answer to lowest terms.

3. Correct Answer: 13^4

○ The student needed to know that exponents represent repeated multiplication. The base (13) is being multiplied by itself four times, so the student would write a raised number 4 to show this.

SKILL: Use an exponent to rewrite an expression with repeated multiplication.

4. Correct Answer: $\sqrt{25}$ or $\sqrt{25} = 5$

○ The student was expected to know a radical is a root. In this case, 5 is the square root of 25, written as shown.

SKILL: Use a radical to show the relationship between a square and its root.

5. Correct Answer: $x = 1.8$

➲ **Sample Solutions:**

1.
$$4x - 11 = -3.8$$
$$(10)4x - 11 = (10)(-3.8)$$
$$40x - 110 = -38$$
$$40x = -38 + 110$$
$$40x = 72$$
$$\frac{40x}{40} = \frac{72}{40}$$
$$x = 1.8$$

2.
$$4x - 11 = -3.8$$
$$4x = -3.8 + 11$$
$$4x = 7.2$$
$$\frac{4x}{4} = \frac{7.2}{4}$$
$$x = 1.8$$

3.
$$4x - 11 = -3.8$$
$$4x = -3.8 + 11$$
$$4x = 7.2$$
$$\frac{1}{4}(4x) = \frac{1}{4}(72)$$
$$x = 1.8$$

➲ The student needed to solve for the unknown (x). He could have solved for the unknown in one of three ways:

➲ **Solution A:**

1. Multiply all terms by 10 so that there are no decimals.

2. Add 110 to both sides to isolate the term with the variable.

3. Divide both sides by the coefficient, 40 (or multiply by the reciprocal, $\frac{1}{40}$).
$$4x - 11 = -3.8$$
$$(10)4x - 11 = (10)(-3.8)$$
$$40x - 110 = -38$$
$$40x = -38 + 110$$
$$40x = 72$$
$$\frac{40x}{40} = \frac{72}{40}$$
$$x = 1.8$$

➲ **Solution B:**

1. Add 11 to both sides to isolate the term with the variable.

2. Divide both sides by the coefficient, 4.
$$4x - 11 = -3.8$$
$$4x = -3.8 + 11$$
$$4x = 7.2$$
$$\frac{4x}{4} = \frac{7.2}{4}$$
$$x = 1.8$$

➲ **Solution C:**

1. Add 11 to both sides to isolate the term with the variable.

2. Multiply both sides of the equation by the reciprocal of the coefficient, $\frac{1}{4}$.
$$4x - 11 = -3.8$$
$$4x = -3.8 + 11$$
$$4x = 7.2$$
$$\frac{1}{4}(4x) = \frac{1}{4}(72)$$
$$x = 1.8$$

SKILL: Solve a two-step equation with one unknown.

6. Correct Answer: $x = 38$

- The student needed to know how to solve for an unknown in a proportion.

- The student could have solved the problem in one of three ways:

- **Solution A:**

  1. Determine the ratio between the two known numerators (in this case, $\frac{2}{3}$).

  2. Multiply the numerator and denominator of the known fraction ($\frac{3}{57}$) by that ratio.

$$^1\cancel{3} \cdot \frac{2}{\cancel{3}_1} = 2 \qquad ^{19}\cancel{57} \cdot \frac{2}{\cancel{3}_1} = 38$$

$$\frac{3}{57} = \frac{2}{38}$$

- **Solution B:**

  1. Determine the ratio between the numerator and denominator of the given fraction (in this case, $\frac{1}{19}$).

  2. Multiply both numerators by the reciprocal of that ratio ($\frac{19}{1}$).

$$\frac{3}{57} = \frac{1}{19}$$

$$3 \cdot \frac{19}{1} = 57 \qquad 2 \cdot \frac{19}{1} = 38$$

$$\frac{3}{57} = \frac{2}{38}$$

- **Solution C:**

  1. Cross multiply the numerators and denominators.

$$\frac{3}{57} = \frac{2}{x} \qquad\qquad \frac{3}{57} = \frac{2}{x}$$

$$3x = 2 \cdot 57 \qquad \textbf{or} \qquad 3x = 2 \cdot 57$$

$$\frac{3}{3}x = \frac{114}{3} \qquad\qquad \frac{1}{3}(3x) = \frac{1}{3}(114)$$

$$x = 38 \qquad\qquad\qquad x = 38$$

  2. Solve for the unknown.

  SKILL: Solve for the unknown in a proportion.

7. Correct Answer: 350 calories

➲ Math-U-See emphasizes the application of mathematical knowledge to solve real-world problems. This particular problem required applying an understanding of ratios and may have resulted in one of the following **Sample Solutions:**

➲ **Solution A:**

1. Write a ratio for the number of crackers in a serving.
2. Multiply by a unit multiplier (a ratio showing the number of calories in a serving) to find the number of calories in one cracker.
3. Multiply the number of crackers by the number of calories in a cracker to arrive at the total.

$$\frac{4 \text{ crackers}}{1 \text{ serving}} \cdot \frac{1 \text{ serving}}{100 \text{ calories}} = \frac{1 \text{ cracker}}{25 \text{ calories}}$$

$$14 \text{ crackers} \cdot \frac{1 \text{ cracker}}{25 \text{ calories}} = 350 \text{ calories}$$

➲ **Solution B:**

1. Write ratios for the number of calories in a serving (4 crackers) and a ratio for the missing information.
2. Set up a proportion with the two ratios and solve. (This sample solution shows solving by cross-multiplying.)

$$\frac{4 \text{ crackers}}{100 \text{ calories}} = \frac{14 \text{ crackers}}{x \text{ calories}}$$

$4x = 14 \cdot 100$

$4x = 1,400$

$x = 350 \text{ calories}$

➲ **Solution C:**

1. Write ratios for the number of calories in a serving (4 crackers) and a ratio for the missing information.
2. Set up a proportion with the two ratios and solve. (This sample solution shows solving by finding the ratio of the known numerators.)

$$\frac{4 \text{ crackers}}{1 \text{ serving}} = \frac{14 \text{ crackers}}{x \text{ servings}}$$

$4x = 14$

Ian ate 3.5 servings.

$3.5 \cdot 100 = 350 \text{ calories}$

SKILL: Use ratio reasoning to solve a word problem.

8. Correct Answer: A matches Y, B matches X, and C matches Z.

➲ This problem assessed the student's spatial reasoning, which is an essential skill for *Geometry*.

SKILL: Identify corresponding sides in congruent figures shown in different orientations.

9. Correct Answer: Quadrant II

⊙ The student needed to know the four quadrants of the coordinate grid, which are identified with Roman numerals.

⊙ In addition, he needed to be able to identify the location of the point after it was moved. The new point is represented as C' in the graphic above. Since the first coordinate is negative and the second is positive, the point would be located in Quadrant II.
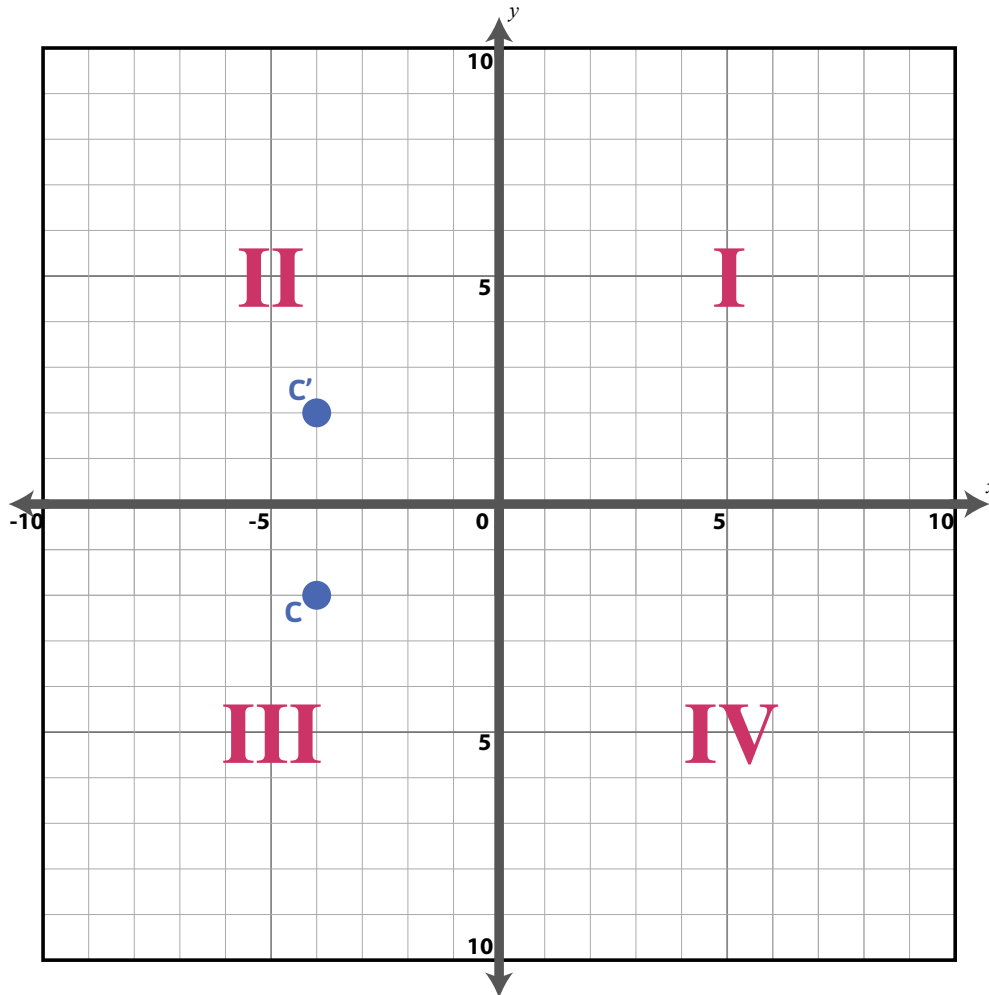
SKILL: Identify the quadrants on a coordinate grid.

10. Correct Answer: He needs to use the 12-cm piece, the 9-cm piece, and the 6-cm piece.

⊙ This particular problem assesses your student's ability to solve a geometric problem logically, without drawing or resorting to trial-and-error. Students using logic to solve this problem would have arrived at these conclusions:

1. Using all four pieces: 12 cm + 10 cm + 9 cm + 6 cm = 37 cm (too long)
Using the longest two pieces: 12 cm + 10 cm = 22 cm (too short)
A combination of three pieces must be used.

2. There were four different possible combinations using three pieces:

» 12 cm, 10 cm, 9 cm

» 12 cm, 10 cm, 6 cm

» 12 cm, 9 cm, 6 cm

» 10 cm, 9 cm, 6 cm

3. Of the four combinations, only one gave a sum of 27 cm.

SKILL: Use logic to solve a real-world problem.

11. A student who has mastered the prerequisite concepts should be able to complete the written assessment in about 45 minutes.

12. A student who has mastered the prerequisite concepts should feel confident in his or her ability to solve the problems and should not need to ask for assistance.