• Analysis •

1. Correct Answer: 15 meters (m)

- The student was expected to know and use the Pythagorean theorem to find the missing side.

  $a^2 + b^2 = c^2$

  $8^2 + x^2 = 17^2$

  $64 + x^2 = 289$

  $289 - 64 = x^2$

  $225 = x^2$

  $\sqrt{225} = x^2$

  $15 = x$

  SKILL: Use the Pythagorean theorem to find the missing side of a right triangle.

2. Correct Answer: $x = -43$

- The student was expected to use knowledge of proportions and solving equations to find the value of x. The sample solutions show two possible ways to solve this problem:

- **Solution A:**
  1. Cross multiply to solve the proportion.
  2. Solve the equation by isolating the variable on one side.

$$\frac{x+3}{5} = \frac{x-5}{6}$$
$$6(x+3) = 5(x-5)$$
$$6x + 18 = 5x - 25$$
$$x + 18 = -25$$
$$x = -43$$

- **Solution B:**
  1. Rewrite the two fractions with common denominators.
  2. Set the numerators equal to one another.
  3. Solve the equation by isolating the variable on one side.

$$\frac{x+3}{5} = \frac{x-5}{6}$$
$$\frac{6x+18}{30} = \frac{5x-25}{30}$$
$$6x + 18 = 5x - 25$$
$$x + 18 = -25$$
$$x = -43$$

SKILLS: Use ratios to solve real-world problems. Solve equations with variables on both sides.

3. Correct Answers: $\frac{a^3}{b^6}$ **or** a^3b^{-6}

- In order to solve this problem, the student needs to have mastered several concepts:

  1. A fraction can be rewritten as a division problem.

     $$\frac{a^5b^{-3}}{b^3a^2} = \left(a^5b^{-3}\right) \div \left(b^3a^2\right)$$

  2. Division by an exponent can be rewritten as multiplication by its opposite.

     $$\left(a^5b^{-3}\right) \div \left(b^3a^2\right) = \left(a^5b^{-3}\right) \cdot \left(b^{-3}a^{-2}\right)$$

  3. Only the same variables can be multiplied together.

     $$\left(a^5b^{-3}\right) \cdot \left(b^{-3}a^{-2}\right) = \left(a^5 \cdot a^{-2}\right) \cdot \left(b^{-3}b^{-3}\right)$$

  4. When variables are multiplied, they can be combined by adding the exponents.

     $$\left(a^5 \cdot a^{-2}\right) \cdot \left(b^{-3}b^{-3}\right) = a^3b^{-6} \text{ or } \frac{a^3}{b^6}$$

  SKILL: Simplify a rational expression.

4. Correct Answer: $8qr - r$

- In order to solve this problem, the student needs to have mastered several concepts:

  1. Variables being multiplied within a term can be combined by adding the exponents.

     $5q^{-1}rq^2 + 3qr - r = 5qr + 3qr - r$

  2. Terms with the exact same variables can be added or subtracted.

     $5qr + 3qr - r = 8qr - r$

  SKILL: Add and subtract to simplify a rational expression.

5. Correct Answer: $\frac{3x+3}{2x}$

- In order to solve this problem, the student needs to have mastered several concepts:

  1. A fraction can be rewritten as a division problem.

     $$\frac{\frac{3}{x}}{\frac{2}{x+1}} = \frac{3}{x} \div \frac{2}{x+1}$$

  2. Division can be rewritten as multiplication by the opposite.

     $$\frac{3}{x} \div \frac{2}{x+1} = \frac{3}{x} \cdot \frac{x+1}{2}$$

  3. To multiply fractions, multiply the numerators together and the denominators together.

     $$\frac{3}{x} \cdot \frac{x+1}{2} = \frac{3(x+1)}{2x} = \frac{3x+3}{2x}$$

  SKILL: Simplify a complex fraction.

6. Correct Answer: $\frac{x^2+2x-6}{x^2-3x+2}$

- In order to solve this problem, the student needed to know the following concepts:

  1. Fractions need to have common denominators before they can be combined.

  The first fraction needed to be multiplied by $\frac{(x-1)}{(x-1)}$,

  and the second fraction needed to be multiplied by $\frac{(x-2)}{(x-2)}$.

  $$\frac{x}{x-2}+\frac{3}{x-1}=$$
  $$\frac{x(x-1)}{(x-2)(x-1)}+\frac{3(x-2)}{(x-2)(x-1)}$$

  2. The Distributive Property needed to be used to create new numerators.

  $$\frac{x(x-1)}{(x-2)(x-1)}+\frac{3(x-2)}{(x-2)(x-1)}=$$
  $$\frac{x^2-x}{(x-2)(x-1)}+\frac{3x-6}{(x-2)(x-1)}=$$

  3. The numerators needed to be combined over the new denominator, and both numerator and denominator needed to be simplified.

  $$\frac{x^2-x+3x-6}{(x-2)(x-1)}=$$
  $$\frac{x^2+2x-6}{(x-2)(x-1)}=$$
  $$\frac{x^2+2x-6}{x^2-3x+2}$$

  SKILL: Add fractions with polynomials in the denominator.

7. Correct Answer: $78\sqrt{2}$

- The student needed to understand how to perform operations with radicals. The sample solutions show two ways in which these concepts could be applied:
- **Solution A:**

Here the radicals are combined first and then simplified.

$$\left(5\sqrt{6}\right)\left(2\sqrt{12}\right)+\left(9\sqrt{8}\right)=$$

$$10\sqrt{72}+9\sqrt{8}=$$

$$10\cdot\left(3\sqrt{8}\right)+9\sqrt{8}=$$

$$30\sqrt{8}+9\sqrt{8}=$$

$$39\sqrt{8}=39\cdot\left(2\sqrt{2}\right)=78\sqrt{2}$$

- **Solution B:**

Here the radicals are simplified first and then combined.

$$\left(5\sqrt{6}\right)\left(2\sqrt{12}\right)+\left(9\sqrt{8}\right)=$$

$$\left(5\sqrt{6}\right)\left(4\sqrt{3}\right)+\left(18\sqrt{2}\right)=$$

$$20\sqrt{18}+9\sqrt{8}=$$

$$60\sqrt{2}+18\sqrt{2}=78\sqrt{2}$$

SKILL: Simplify expressions with radicals.

8. Correct Answer: $\frac{3}{2}$

- In order to solve this problem, the student needs to have mastered several concepts:

1. A negative exponent indicates the reciprocal of the base.

$$\left(\frac{8}{27}\right)^{-\frac{1}{3}}=\left(\frac{27}{8}\right)^{\frac{1}{3}}$$

2. A fractional exponent indicates a root. In this case, the exponent $\left(\frac{1}{3}\right)$ means the cube root of the base.

$$\left(\frac{27}{8}\right)^{\frac{1}{3}}=\sqrt[3]{\left(\frac{27}{8}\right)}\text{ or }\frac{\sqrt[3]{27}}{\sqrt[3]{8}}$$

3. The cube root is a number that it is multiplied by itself three times to obtain the given number. In this case, 3 · 3 · 3 = 27, and 2 · 2 · 2 = 8.

$$\sqrt[3]{\left(\frac{27}{8}\right)}\text{ or }\frac{\sqrt[3]{27}}{\sqrt[3]{8}}=\frac{3}{2}$$

SKILL: Simplify an expression with a fractional exponent.

9. Correct Answer: 4

- SAMPLE SOLUTIONS:

1. $\dfrac{2\sqrt{28}}{\sqrt{7}} = \dfrac{2\sqrt{28}}{\sqrt{7}} \cdot \dfrac{\sqrt{7}}{\sqrt{7}} = \dfrac{2\sqrt{196}}{\sqrt{49}} = \dfrac{2 \cdot 14}{7} = \dfrac{28}{7} = 4$

2. $\dfrac{2\sqrt{28}}{\sqrt{7}} = 2\sqrt{\dfrac{28}{7}} = 2\sqrt{4} = 2 \cdot 2 = 4$

SKILL: Simplify a division expression with irrational numbers.

10. Correct Answer: $(x^3 - 4b)\ (x^3 + 4b)$

- The student should have recognized the expression as the difference of two squares and factored it accordingly.

SKILL: Factor the difference of two squares.

11. Correct Answer: -72

- The student needed to know that i is an imaginary number representing the square root of -1. He then needed to apply this information to rename square roots of negative numbers and simplify expressions, as follows:

$$(9i)\left(\sqrt{-64}\right) = (9i)\left(\sqrt{-1}\right)\left(\sqrt{64}\right) = (9i)(i)\left(\sqrt{64}\right) = \left(9i^2\right)\left(\sqrt{64}\right) = (9)(-1)(8) = -72$$

SKILL: Simplify expressions with imaginary numbers.

12. Correct Answers: $\dfrac{3-3\sqrt{3}}{2}$ **or** $\dfrac{1}{2}\left(3-3\sqrt{3}\right)$

- The student was expected to know that he needed to multiply the denominator by the conjugate $(1-\sqrt{3})$ in order to complete the square and obtain a whole-number denominator.

$$\frac{3}{1+\sqrt{3}} = \frac{3}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{3(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{3(1-\sqrt{3})}{1-3} = \frac{3(1-\sqrt{3})}{2} \text{ or } \frac{3}{2}(1-\sqrt{3})$$

SKILL: Rationalize the denominator of a fraction.

13. Correct Answer:

$$x = \frac{5 \pm \sqrt{17}}{2}$$

or

$$x = \frac{5+\sqrt{17}}{2}, \frac{5-\sqrt{17}}{2}$$

- The student was expected to notice that the equation could not be solved by factoring; therefore, he needed to know and use the quadratic formula to find the solution.

$$x^2 - 5x = -2$$
$$x^2 - 5x + 2 = 0$$
$$\text{Quadratic formula: } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{5 \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1}$$
$$x = \frac{5 \pm \sqrt{25 - 8}}{2}$$
$$x = \frac{5 \pm \sqrt{17}}{2}$$

SKILL: Use the quadratic formula to solve an equation.

14. Correct Answer: E; $x^2 + y^2 = 16$

- The student was expected to know the general equation for a circle: $(x-h)^2+(y-k)^2= r^2$
- The student should also know the general equations for the other answer choices:

A. $y = 3x$

- » This is the general equation of a line: $y = mx + b$
- » This equation can be rewritten as: $y = 3x + 0$

B. $y^2 = 4x^2 - 12$

- » This is the general equation of a hyperbola : $x^2 - y^2 = c$
- » This equation can be rewritten as: $4x^2 - y^2 = -12$

C. $y = \frac{4}{x}$

- » This is also the general equation of a hyperbola: $xy = c$
- » This equation can be rewritten as: $xy = 4$

D. $y = x^2 - 25$

- » This is the general equation of a parabola: $y = x^2$

SKILL: Identify the shape of a graph from its equation.

15. Correct Answer: $\frac{5\text{ cm}}{1\text{ mo}} \times \frac{12\text{ mo}}{1\text{ yr}} \times \frac{1\text{ m}}{100\text{ cm}} = \frac{3}{5}\text{m/yr}$ (or 0.6 m/yr)

- The glacier moves 0.6 meters in a year.

SKILL: Use unit multipliers (dimensional analysis) to convert units.

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16. A student who has mastered the prerequisite concepts should be able to complete the written assessment in about 45 minutes.

17. A student who has mastered the prerequisite concepts should feel confident in his or her ability to solve the problems and should not need to ask for assistance.