• Analysis •

1. Correct Answers: $\dfrac{-3x-45}{x^2-25}$ or $-\dfrac{3x+45}{x^2-25}$

- **Sample Solution:**

$$\frac{2}{x+5}+\frac{5}{5-x}-\frac{10}{x^2-25}=$$
$$\frac{2}{x+5}-\frac{5}{x-5}-\frac{10}{(x+5)(x-5)}=$$
$$\frac{2(x-5)}{(x+5)(x-5)}-\frac{5(x+5)}{(x-5)(x+5)}-\frac{10}{(x+5)(x-5)}=$$
$$\frac{2x-10}{(x+5)(x-5)}-\frac{5x+25}{(x-5)(x+5)}-\frac{10}{(x+5)(x-5)}=$$
$$\frac{2x-10}{(x+5)(x-5)}+\frac{-5x-25}{(x-5)(x+5)}+\frac{-10}{(x+5)(x-5)}=$$
$$\frac{2x-10-5x-25-10}{(x+5)(x-5)}=$$
$$\frac{-3x-45}{(x+5)(x-5)}$$

- At this level, students are expected to be skilled at simplifying rational expressions. For this particular problem, the student needed to have mastered the following concepts:
  - Factor the difference of two squares
  - Identify the effect of a negative sign on an entire expression
  - Rename rational expressions so that they have a common denominator
  - Simplify expressions by combining like terms

  SKILL: Simplify a rational expression.

2. Sample Answers:

The graph of $2x^2$ is narrower than the graph of x^2.

The graph of x^2 is wider than the graph of $2x^2$.

- Ideally, the student should have been familiar with the standard equation of a parabola ($y = x^2$) and should not have needed to graph the equations. He should have known that adding a coefficient of 2 to the term x^2 would make the graph narrower.

  SKILL: Identify the effect of the coefficient on the graph of a second-degree equation.

3. Correct Answer: The x and y axes are the asymptotes.

- Is the student's answer correct?
  - Yes
  - No
- The student needed to know that an asymptote is a line that a graph approaches but never meets. In this case, both parts of the graph come close to both the x axis and y axis but will never meet them. This is because x and y will never equal zero.

  SKILL: Identify the asymptote(s) for a hyperbola.

4. Correct Answer: (3, -8)

- The student should have worked through either of the following solutions to find the vertex (in this case, the lowest point of the parabola).
- **Sample Solution 1:**
  1. Compare the equation to the standard form of a quadratic: $y= ax^2 + bx + c$.
     For this equation, $a = 1$, $b = -6$, and $c = 1$.
  2. The x value of the vertex is found by using the formula $\frac{-b}{2a}$. In this case, $x = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$.
  3. Substitute this value into the equation to find the value of y.
  4. $y=$ (3)2 – 6(3) + 1 = 9 – 18 + 1 = -8
  5. The vertex is located at (3, -8).
- **Sample Solution 2:**
  1. Complete the square to rewrite the equation in vertex form of a quadratic: $y= a(x – h)2 + k$.
     In this case, something must be added to $x^2 – 6x$ to make it the perfect square of a polynomial.
     You can add the zero pair of 9 and -9 to accomplish this purpose.
  2. $y= x^2 – 6x + 1$
  3. $y= (x^2 – 6x + 1) + (9 – 9)$
  4. $y= (x^2 – 6x + 9) + (1 – 9)$
  5. $y= (x – 3)2 – 8$
- The vertex is located at (h, k), or, in this case, (3, -8).

  SKILL: Identify the vertex of a quadratic.

5. Correct Answer: $\frac{5}{3}$, 1.67, or $1.6\overline{6}$ (Do not accept this answer as $1\frac{2}{3}$.)

- First, the student needed to identify the 4-m side of the triangle as the opposite side, the 3-m side as the adjacent side, and/or the 5-m side as the hypotenuse. Then he should have known that the secant is the reciprocal (inverse, or opposite) of the cosine, or the ratio of the measures of the hypotenuse and the adjacent side in the triangle.

  This ratio is $\frac{5}{3}$, which can be calculated as 1.6667 or $1.666\overline{6}$.

  SKILL: Identify the reciprocals of the basic trigonometric functions.

6. Correct Answer: 40°

- **Solution:**
  - » The student should have begun at the x-axis and moved 320° clockwise because the sign is negative.
  - » From there, the student should have drawn a perpendicular line from the terminal side to the x-axis.
  - » The measure of that angle is 40°.

  SKILL: Find the reference angle for any given angle.

7. Correct Answer: $\frac{2-\sqrt{3}}{2}$ or $\frac{-\sqrt{3}+2}{2}$

- **Sample Solution:**

$$-\frac{\sqrt{3}}{2}+2\left(\frac{1}{2}\right)=-\frac{\sqrt{3}}{2}+1=-\frac{\sqrt{3}}{2}+\frac{2}{2}=\frac{2-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}+2}{2}$$

- The student should know the values for common trigonometric ratios. In this case, he should have known that the sine of 240° is $-\frac{\sqrt{3}}{2}$ and the cosine of 300° is $\frac{1}{2}$. He would then insert this information into the expression and simply accordingly.

  SKILL: Identify the ratios for common trigonometric functions.

8. Correct Answers: $\cos^2(x)$ or $\frac{1+\cos 2x}{2}$

- The student should know the basic trigonometric identity, $\sin^2(x) + \cos^2(x) = 1$. This would have enabled him to rewrite the expression as $1 - \sin^2(x) = \cos^2(x)$. Some students may also be able to recall the double-angle identity $\sin 2(x) = \frac{1-\cos 2x}{2}$. This could be substituted into the expression $1 - \sin^2(x)$ as follows:

$$1-\frac{1-\cos 2x}{2}=\frac{2}{2}-\frac{1-\cos 2x}{2}=\frac{2-(1-\cos 2x)}{2}=\frac{2-1+\cos 2x}{2}=\frac{1+\cos 2x}{2}$$

  SKILL: Identify the basic trigonometric identities.

9. Correct Answer: $\frac{180}{\pi}$

- The student should know the relationship between degrees and radians and should be able to convert one to the other.

  SKILL: Convert between degrees and radians.

10. Correct Answers: The range includes all values greater than or equal to 0.

$f(x) > 0$

$y > 0$

$[0, \infty)$

$\{y / y > 0\}$

$\{f(x) / y > 0\}$

- The student should know that the range refers to all possible values for $f(x)$. In this case, because the function involves a radical, the value of the expression inside the radical $(x + 3)$ can never be less than zero. This information can be presented in any of the ways shown as correct answers. (Note: the student may not have read carefully and may have identified the domain (all possible values of x) as all numbers greater than or equal to negative 3.)

  SKILL: Identify the range of a function.

11. Correct Answers:

$(2x + 3)^2$

$(2x + 3)(2x + 3)$

$4x^2 + 12x + 9$

- The student should have seen that he needed to substitute the entire expression for f(x), $2x + 3$, into the g(x) function, x^2. This would have given any of the correct answers shown.

  SKILL: Evaluate composite functions.

12. Correct Answers: x = -4 or 1; x = {-4, 1}

- The student should be familiar with logarithms and know how to manipulate them. In this case, because both logarithms have a base of 2, they can be combined by multiplication. The student should also have known that a logarithm with a base of 2 can be renamed as an exponent of 2, which would have enabled the student to solve as follows:

  $\log_2 x + \log_2 (x + 3) = 2$

  $\log_2 x (x + 3) = 2$

  $x (x + 3) = 2^2$

  $x^2 + 3x = 4$

  $x^2 + 3x - 4 = 0$

  $(x + 4)(x - 1) = 0$

  $x + 4 = 0$ or $x - 1 = 0$

  $x = -4$ or $x = 1$

  $x = \{-4, 1\}$

  SKILL: Simplify logarithmic expressions.

13. Correct Answer: 0.8047 (If the student did not have a calculator, accept $\frac{\ln 5}{2}$.)

- The student should be familiar with natural exponential and logarithmic functions and be able to manipulate them. This would have enabled him to solve the equation as follows:

$$e^{2x} = 5$$
$$\ln(e^{2x}) = \ln 5$$
$$2x(\ln e) = \ln 5$$
$$2x = \ln 5$$
$$x = \frac{\ln 5}{2}$$
$$x = \frac{1.6094}{2}$$
$$x = 0.8047$$

SKILL: Solve exponential equations.

14. Sample Answers: (Accept either or both):

The period of $y = \sin 2(x - \frac{\pi}{4})$ is π, while the period of $y = \sin x$ is 2π.

The graph of $y = \sin 2(x - \frac{\pi}{4})$ moves to the right of the graph of $y = \sin x$; OR

The graph of $y = \sin 2(x - \frac{\pi}{4})$ has a horizontal shift to the right of $\frac{\pi}{4}$.

- The student needed to be familiar with the parent graphs for the basic trigonometric functions—in this case, the sine function. The student should have identified the two differences between the equations and interpreted how they would affect the graph. The sine graph normally has a period of 2π, which would be reduced by half because of the coefficient of 2 in the other equation. Also, subtracting would move the graph of the sine function to the right by that amount.

  SKILL: Identify how the characteristics of an equation affect the standard graph of a trigonometric function.

15. Correct Answer: -4

- The student should have been able to evaluate the limit as follows:

$$\lim_{x \to 1} 2x^2 - 6$$

$$2(1)^2 - 6 = 2(1) - 6 = 2 - 6 = -4$$

  SKILL: Evaluate a limit.